TONBRIDGE SCHOOL

Scholarship Examination 2014

MATHEMATICS II

Wednesday 30th April 2014
2.00 pm

Time allowed: 1 hour 30 minutes

There are seven questions in this paper
Answer as many questions as you can.
All the questions carry equal marks.
You may attempt the questions in any order.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. A formula due to Einstein that occurs in physics is $B = A\sqrt{1 - \frac{v^2}{c^2}}$. In the following parts give your answers as decimals rounded correct to one decimal place.

   (a) If $A = 10$, $v = 2.9$ and $c = 3$, find B.

   (b) If $B = 23$, $v = 2.5$ and $c = 3$, find A.

   (c) If $B = 35$, $A = 35.9$ and $c = 3$, find v.

   (d) If $A = 56$, $B = 27.1$ and $v = 2.8$ find c.

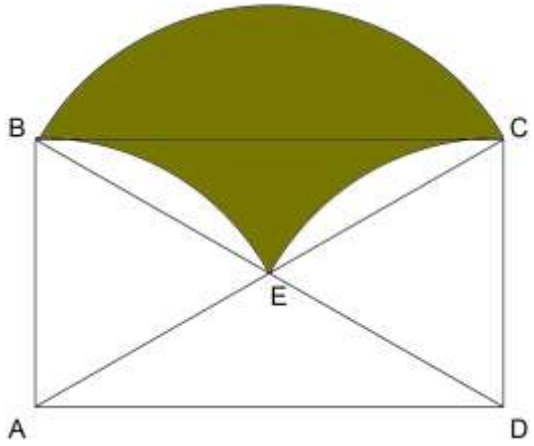
2. (a) Find the solution of the simultaneous equations below:

$$a + b + c = 15$$
$$a + b - c = 5$$
$$a - b + c = 11$$

   (b) Find *all* the solutions of the simultaneous equations below:

$$xy - \frac{2x}{y} = 6$$
$$3xy - \frac{5x}{y} = 21$$

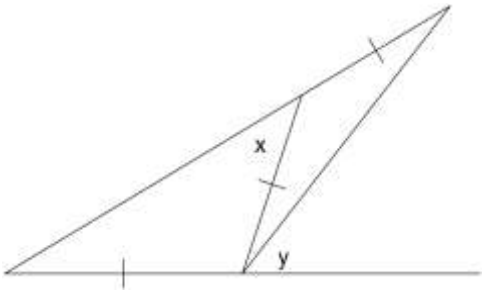
3. The figure below shows a rectangle *ABCD* with diagonals *AEC* and *BED*. Triangles *ABE* and *CDE* are equilateral with all of their sides 10 cm long. The arc from *B* to *E* is part of a circle with centre *A*, the arc from *E* to *C* has centre *D* and the arc from *B* to *C* has centre *E*.



   (a) Find the area of the unshaded part of rectangle *ABCD*.

   (b) Find the area of the shaded region.

4. In the diagram below the marked lengths are all equal and the long lines at the bottom and side of the figure are both straight.



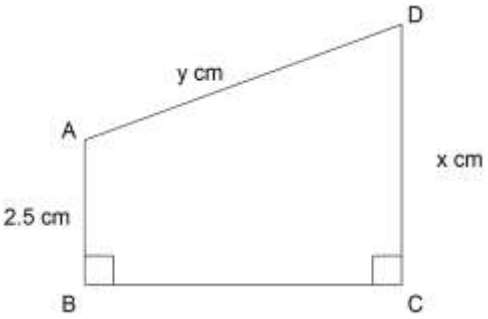
(a) (i) If $x = 50°$, find y.

(ii) If $x = 80°$, find y.

(b) Use algebraic reasoning on the figure to find y in terms of x: you should simplify your answer as far as you can.

(c) What is the maximum possible value of y in the diagram? (It is not 180°!)

5. The figure below is a trapezium in which vertices (corners) *A*, *B*, *C* are fixed throughout the question, but *D* is free to move up and down vertically. The connection between length y and length x is given by the formula $y = \sqrt{x^2 - 5x + 8}$.



(a) Find the values of y for $x = 0, 1, 2, 3, 4, 5, 6, 7$.

(b) By using these values and choosing sensible scales on both the *x*- and *y*-axes, plot a neat graph of y against x.

(c) Explaining your method carefully, use your graph to find the length of *BC*.

(d) Use the diagram above to explain why your graph has a vertical axis of symmetry.

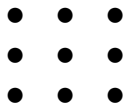
TURN OVER

6. In this question you will need to use the fact that a solid sphere of radius r has surface area (S) given by the formula $S = 4\pi r^2$ and volume (V) given by the formula $V = \frac{4}{3}\pi r^3$.

(a) If 50 solid metal spheres each of radius 6 cm are melted together and reshaped into a solid cube, what is the side-length of the cube?

(b) If 50 solid metal cubes each of side-length 6 cm have all of their faces painted, what is the radius of the single solid metal sphere that could be covered with the same amount of paint?

7. The diagram shows a square grid of dots with 3 dots in each row and column.



(a) Use diagrams to explain why there are 6 squares that can be drawn with their four corners on the grid: their sides do *not* have to be horizontal or vertical.

The table below shows data for other square grids of dots. Column A shows the number of dots in each row and column of the grid; Column B shows the total number of squares that can be drawn with their corners on the grid.

	A	**B**	**C**	**D**
Row 1	1	0	0	1×0
Row 2	2	1	12	4×3
Row 3	3	6	72	9×8
Row 4	4	20	240	16×15
Row 5	5	50	600	25×24
Row 6				
Row 7				
Row *n*				

(b) Write down the entries that would be in Columns A, B, C, D for Row 6 and Row 7.

(c) Give formulae (in terms of n) for the entries in Columns D and B for Row n.

(d) Find the entry in Column B for Row 20.

(e) What is the entry in Column A if the entry in Column B is 192660?

END OF PAPER